Different eigenproblem models for field line resonances in cold plasma: Effect on magnetospheric density estimates

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Abstract

Magnetospheric plasma density can be remotely sensed through ground-based magnetometer data using a suitable model for field line resonances (FLRs) formed by standing shear Alfven wave on closed geomagnetic field lines. The simplest type of FLR model, which is also the most relevant for magnetometer data inversion purposes, is based on solving a certain eigenvalue problem. Over the years a number of such models have been developed [Singer, H.J., Southwood, D.J., Walker, R.J., Kivelson, M.G., 1981. Alfven wave resonances in a realistic magnetospheric magnetic field geometry. J. Geophys. Res. 86, 4589–4596; Rankin, R., Fenrich, F., Tikhonchuk, V.T., 2000. Shear Alfven waves on stretched magnetic field lines near midnight in Earth’s magnetosphere. Geophys. Res. Lett. 27, 3265–3268; Rankin, R., Kabin, K., Marchand, R., 2006. Alfvenic field line resonances in arbitrary magnetic field topology. Adv. Space Res. 38, 1720–1729]. In this paper we summarize the properties of these models and investigate the effect of using these different models on the magnetospheric density inferred from the ground-based magnetometer measurements. We also formulate a simple criterion which can be used to determine which one of these models should be used for a particular field line.

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1. Introduction

Ground-based magnetometer measurements in the ultra-low frequency (ULF) range are increasingly used to estimate the magnetospheric plasma density (Waters et al., 1996; Menk et al., 1999; Denton and Gallagher, 2000; Dent et al., 2003; Clilverd et al., 2003; Rankin et al., 2005). These techniques provide a very convenient way for sensing remotely the magnetosphere and yield results which can be later used in conjunction with satellite measurements for creating multidimensional maps of density distribution in the Earth magnetosphere. The accuracy of such density estimates, however, critically depends on three factors: the parametric plasma density distribution model, the background magnetic field model, and the model for standing shear Alfven waves along magnetic field lines. Below we briefly outline these three causes of uncertainty.

The plasma density is commonly assumed to be distributed according to a power law (e.g. Cummings et al., 1969; Orr and Matthew, 1971):

\[ \rho = \rho_{eq}(L) \left( \frac{R_E L}{r} \right)^m, \]  

where \( R_E \) is the Earth radius, \( L \) is the \( L \)-shell, and \( r \) is the distance from the center of the Earth. The value of the exponent is often taken to be \( m = 4 \) (e.g. Waters et al., 1996; Dent et al., 2003), although some satellite measurements suggest a smaller value of \( m = 1 - 2 \) (Denton et al., 2002). Parametric studies using different values of \( m \) (e.g. Orr and Matthew, 1971) have shown that this parameter has a relatively small effect (typically, less than 10%) on
the frequencies of the FLRs. Once this parametric form is assumed, the only remaining unknown parameter is \( \rho \), the equatorial plasma density for a particular field line, which can be determined from the measured FLR frequency. More sophisticated multi-parametric density models were also considered, for example by Denton et al. (2001). In this case, however, the determination of the density profile parameters requires knowledge of the frequencies of several harmonics in addition to the fundamental frequency.

In contrast to the details of the density distribution, the background magnetic field model can have a very significant effect on the plasma density estimation, as shown, for example, by Rankin et al. (2000), Lui and Cheng (2001), and Waters et al. (2006). Field lines described by realistic magnetic field models, which account for the magnetotail stretching on the nightside, resonate at much lower frequencies (sometimes, by an order of magnitude) than is the case for simple dipole field lines. We note, however, that it is very difficult to assess the accuracy of empirical background magnetic field models. Some estimations of the accuracy of Tsyganenko magnetic field models are given, for example, by Tsyganenko (2002) who shows that for quiet and moderately disturbed magnetospheric conditions, the accuracy of these models is better on the dayside, than on the night side and usually the \( B_x \) and \( B_z \) components are reproduced better than the \( B_y \) component. Typical errors of Tsyganenko models in the outer magnetosphere are on the order of 5 nT for all magnetic field components, when compared to satellite observations. It is, however, quite difficult to estimate the errors in the magnetic field lines arising from inaccuracies of the background magnetic field model.

The third cause of uncertainty is related to how sensitive the density estimates are to the various models for the cold plasma FLRs. This cause has not been studied in the past, and it is the focus of the present paper. For a dipole magnetic field, the eigenproblem approach to computing a spectrum of FLR frequencies is unique and well known (e.g. Cummings et al., 1969). For non-dipole fields, however, three different eigenproblem formulations have been introduced by Singer et al. (1981), Rankin et al. (2000), and Rankin et al. (2006), respectively, for estimating the FLR frequencies. The details of these three approaches are described in Section 2. We note that the ultimate test of any remote sensing procedure is, of course, a comparison with in situ measurements. However, such a test will lump together all errors inherent in the technique, regardless of their origin, while in this paper we focus on a specific cause of the uncertainty. We feel that an analysis of all individual blocks of a technique is necessary before it is widely applied.

In the past, more advanced multidimensional models for FLRs have been developed, which include plasma temperature and dispersion effects (e.g. Streltsov, 1999; Proehl et al., 2002; Cheng, 2003; Lu et al., 2003; Rankin et al., 2004). These models generally require an initial equilibrium distribution of plasma, which is usually not known, and are more suitable for investigating waves appearing in the magnetosphere in response to particular external drivers than to evaluating the eigenfrequencies of specific field lines. Therefore, these models are not well adapted for probing magnetospheric density for specific events. Other models including warm plasma effects, such as Klimushkin et al. (2004), allow a reasonably straightforward calculation of FLR frequencies for specific field lines, but still require essentially unconstrained pressure profiles as inputs. These models, however, show that the radial pressure gradient can have a strong effect on the frequency of poloidal mode (Denton et al., 2003). Since this gradient is often unknown, this makes magnetospheric density estimations based on the poloidal mode less reliable compared with those based on the toroidal mode.

In the present paper we compare the results of three cold plasma eigenproblem models (Singer et al., 1981; Rankin et al., 2000, 2006) which have as few adjustable parameters as possible. These models can utilize either empirical or global Magnetohydrodynamics background magnetic field models, require very modest computational resources and are a natural choice for remote sensing the density content of the magnetosphere in the absence of detailed information about plasma pressure distribution in the magnetosphere.

## 2. Different eigenproblem models for field line resonances

The most widely used approach to date for computing FLR frequencies in a non-dipole topology is based on the equation of Singer et al. (1981) for the plasma displacement \( \xi \) in a certain direction perpendicular to the magnetic field line:

\[
\frac{d^2}{ds^2} \left( \frac{\xi}{h_a} \right) + \frac{d}{ds} \ln(h_a) B_0 \frac{d}{ds} \left( \frac{\xi}{h_a} \right) + \frac{\omega^2}{v_A^2} \left( \frac{\xi}{h_a} \right) = 0. \tag{2}
\]

Here \( s \) is the distance along the field line, \( v_A = B_0 / \sqrt{\rho_0} \) is the Alfvén speed, \( B_0 \) is the background magnetic field, and \( h_a \) is a geometric scale factor calculated as follows. At some point along the field line (usually, at one of the ionospheres or at the magnetic equator) a vector \( a \) normal to the magnetic field line is chosen and an auxiliary field line separated at this point from the main one by a small distance in the \( a \) direction is traced. Then, \( h_a \) is set to be proportional to the distance between the main and auxiliary field lines at every point. Of course, the determination of how close the auxiliary field line should be located to the main one may require some trial and error. In addition, there are numerical difficulties associated with the calculation of the distance between the two field lines, due to the fact that they are, in general, not parallel to one another. The direction at every point from the main field line to the auxiliary one defines the direction of the plasma displacement at this point (which is also the polarization of the wave).
The approach of Singer et al. (1981) is rigorously justified if both the main and the auxiliary field lines are coordinate lines (corresponding to constant values of some coordinate perpendicular to the magnetic field direction) lying in the same coordinate plane. This assumption, however, is equivalent to the requirement that an orthogonal field aligned coordinate system exists, which is unfortunately not true in general. Salat and Tataronis (2000) discuss the conditions for existence of such a coordinate system and show that it does not exist for virtually any non-axisymmetric magnetic field. Therefore, the model of Singer et al. (1981) is not self-consistent when applied to realistic magnetospheric magnetic fields. Another somewhat unsatisfactory aspect of the Singer et al. (1981) model is that it can be formally applied to an arbitrary polarization of the wave as there are no restrictions on the direction of \( \mathbf{a} \). Therefore, the polarization of a wave described by equation (2) will change along the main field line simply following the direction toward the (arbitrarily chosen) auxiliary field line, rather than being determined by physical considerations. However, despite its limitations, the method of Singer et al. (1981), has been used extensively over the years and has proven to be a useful practical approximation in many situations (e.g. Waters et al., 1996; Dent et al., 2003).

Another simple model for computing the FLR frequencies was derived by Rankin et al. (2000); it can be considered as simply a modification of the model of Singer et al. (1981), which comes from a realization that in an axisymmetric magnetic field topology, the scale factors are given explicitly by \( h_1 \sim R \) and \( h_2 \sim (RB_0)^{-1} \) where \( R(s) \) is the distance from a point on the field line to the symmetry axis. This observation makes a calculation using an auxiliary field line unnecessary. In Rankin et al. (2006), as well as here, we propose to use this model in general magnetic fields as an approximation and study the limits of its applicability. The equations of Rankin et al. (2000) for toroidal (\( B_1 \)) and poloidal (\( B_2 \)) magnetic field perturbations can be written explicitly as:

\[
\begin{align*}
\frac{d^2(h_1 B_1)}{ds^2} + \frac{d(h_1 B_1)}{ds} \cdot \frac{d}{ds} \ln \left( \frac{B_1^2 R^2}{\rho} \right) + \frac{\omega^2}{v_A^2} (h_1 B_1) &= 0, \quad (3) \\
\frac{d^2(h_2 B_2)}{ds^2} + \frac{d(h_2 B_2)}{ds} \cdot \frac{d}{ds} \ln \left( \frac{B_2^2 R^2}{\rho} \right) + \frac{\omega^2}{v_A^2} (h_2 B_2) &= 0. \quad (4)
\end{align*}
\]

The boundary conditions are \( d(h_{1,2} B_{1,2})/ds = 0 \) at both ionospheres, assuming infinite ionspheric conductivity. Obviously, the same problem can be formulated in terms of electric field perturbations or plasma displacement, but we do not need these equations in this paper. The relation between the electric and magnetic field components of the wave can be obtained from Faraday’s law and are given, for example, in Rankin et al. (2006). Note that the model of Rankin et al. (2000) is only applicable to the two polarizations which were explicitly assumed in the derivation of Eqs. (3) and (4).

Eqs. (3) and (4) are exact in axisymmetric magnetic field topology, which is also the case when an orthogonal field aligned coordinate system exists (Salat and Tataronis, 2000) and the model of Singer et al. (1981) is applicable. In this case, the two models are equivalent; however, the approach of Rankin et al. (2000) has a certain numerical advantage over the method of Singer et al. (1981) because information is required along a single field line only. However, when formally applied to a more general magnetic field configurations, neither model is exact, and they produce somewhat different results.

A more rigorous approach to FLR modeling in a general magnetic field topology is to define a non-orthogonal field-aligned coordinate system based on the Euler potentials (e.g. Stern, 1970; D’haeseleer et al., 1991), and to compute the corresponding metric tensor (which is in general non-diagonal) numerically, as described in Rankin et al. (2006). Then, the magnetic and electric field perturbations have to be described in terms of their covariant (or contravariant) components \( \delta B_{1,2} \), \( \delta E_{1,2} \) and the linearized equations for shear Alfven waves in cold plasma can be written as

\[
\begin{align*}
\frac{1}{\sqrt{g}} \frac{\partial \delta B_1}{\partial u} &= \frac{1}{v_A^2} (g^{11} \omega \delta E_1 + g^{12} \omega \delta E_2), \\
\frac{1}{\sqrt{g}} \frac{\partial \delta B_2}{\partial u} &= \frac{1}{v_A^2} (g^{21} \omega \delta E_1 + g^{22} \omega \delta E_2), \\
\frac{1}{\sqrt{g}} \frac{\partial \delta E_1}{\partial u} &= -(g^{12} \omega \delta B_1 + g^{22} \omega \delta B_2), \\
\frac{1}{\sqrt{g}} \frac{\partial \delta E_2}{\partial u} &= (g^{11} \omega \delta B_1 + g^{12} \omega \delta B_2).
\end{align*}
\]

Here \( u \) is the field-aligned coordinate, \( g^{11}, g^{12}, \) and \( g^{22} \) are the components of the metric tensor and \( g \) is the determinant of the inverse tensor. In the special case when the system of coordinates is orthogonal, the components of the metric tensor are related to the scale factors as \( g^{11} = 1/h_1^2, \ g^{22} = 1/h_2^2, \ g^{12} = 1/h_3^2 \), \( g^{13} = g^{23} = g^{33} = 0 \), and \( \sqrt{g} = h_1 h_2 h_3 \).

Eqs. (5)–(8) form a system of four coupled first order differential equations describing coupled poloidal and toroidal modes in the case of a general background magnetic field geometry. The frequency and the polarization of the wave have to be found from an eigenvalue problem for these equations with the boundary conditions \( \delta E_1, \delta E_2 = 0 \) (assuming high ionospheric conductivity) at both ends of the field line. Note that, unlike the previous two models, the polarization of the wave is now obtained in a self-consistent manner, rather than being externally imposed. The most straightforward way to solve this eigenvalue problem is by using a shooting method (Press et al., 1992, Chapter 17). As a part of the shooting method we have to find a root of a system of two equations (for the frequency and polarization) which can be accomplished by using a multidimensional Newton’s method (Press et al., 1992, Chapter 9). Unfortunately, Newton’s method may sometimes converge to some high
harmonic (or even fail to converge). Thus, the robustness of the Rankin et al. (2006) model is still somewhat of an issue. Results presented in this paper often required numerous modifications to the initial guesses for the frequency and polarization of the wave before convergence to the desired root was achieved.

It should be noted that this eigenvalue problem is different from the classical Sturm–Liouville problem because we have four first-order equations and two parameters (frequency \( \omega \) and the polarization of the wave at the northern ionosphere) instead of a single second order equation with a single parameter. Properties of multi-parameter eigenvalue problems are still actively researched in mathematics (e.g. Faierman, 1991), and we are unaware of any general theorems which apply to our particular form of the two-parameter eigenvalue problem. Although for some particularly stretched and twisted field lines the convergence of the newton iterations was quite slow, in general we were able to identify the fundamental modes with our numerical scheme, even if a rigorous mathematical analysis of the problem is still lacking.

3. Density estimates with different eigenproblem models

In this paper, we compare the three different eigenproblem formulations described in Section 2 using the CANOPUS magnetometer measurements taken between 08:00 UT February 9, 1995 and 04:00 UT February 10, 1995 as an example. This event was analyzed in detail by Waters et al. (2006) who used cross-phase analysis (Waters et al., 1994) to determine the frequencies of standing Alfvén waves at the midpoints between the Churchill line magnetometer stations. Magnetic activity during the observation period was fairly low \( (DST = -16, Kp \approx 0) \), so this event can be viewed as a typical quiet day. Fig. 1 shows the fundamental mode frequencies deduced from the data by Waters et al. (2006) and the corrected geomagnetic (CGM) coordinates corresponding to the midpoints of the Churchill line magnetometer pairs. The horizontal axis on this plot is the magnetic local time (MLT) on the ground at the moment of the observation. For consistency with Waters et al. (2006) we use Tsyganenko 96 (Tsyganenko, 1995; Tsyganenko and Stern, 1996) as the magnetic field model and the power-law density distribution with \( m = 4 \).

The Tsyganenko 96 model is initialized with the same conditions as those used by Waters et al. (2006), specifically \( P_{\text{dy}} = 1.85 \, \text{nPa}, \quad DST = -16, \quad \text{Interplanetary Magnetic Field (IMF)} \quad B_y = 4.91 \, \text{nT} \quad \text{and} \quad B_z = 4.64 \, \text{nT} \). Although the solar wind and IMF conditions were changing somewhat during the observation period, these changes were not dramatic and the above values provide good average estimates for the magnetospheric conditions.

Fig. 2 shows the density estimated with the Singer et al. (1981) model assuming toroidal (bold solid line) and poloidal (bold dashed line) polarizations at the ionosphere (with respect to the dipole axis). Results computed with the Rankin et al. (2000) model are presented with thin solid lines for the toroidal model and thin dashed lines for the poloidal model. The toroidal-like mode of the Rankin et al. (2006) model is shown in Fig. 2 with a dotted line and the poloidal-like mode with a dash-dotted line. Note that for consistency with Waters et al. (2006) the horizontal axis of the plot shows the MLT in the equatorial plane (defined by \( Z_{\text{GSM}} = 0 \)). The maximum distances from the center of the Earth for the field lines at different MLTs (an \( L \) shell-like parameters) are shown as a second line under the horizontal axis in the panels of Fig. 2.

The results of all three approaches agree quite well for both toroidal and poloidal modes for all stations except the two highest latitudes: Rankin–Eskimo and Eskimo–Fort Churchill. This good agreement for low latitudes is not surprising, since the corresponding field lines are expected to be close to dipole field lines and approximately axisymmetric. Therefore, low CGM latitudes fall into the domain of validity of both Singer et al. (1981) and Rankin et al. (2000) models. We further note that at low latitudes the results of Rankin et al. (2000) seem to agree better than those of Singer et al. (1981) with the more accurate model of Rankin et al. (2006). We feel that this disagreement is most likely the result of the inaccuracies of the finite-difference approach used by Singer et al. (1981) model to compute the scale factors. The disagreement between the models at higher latitudes arises from the deviation of the magnetic field lines from even approximate axial symmetry. This, however, invalidates the assumptions behind the models of either Singer et al. (1981) or Rankin et al. (2000). We see no reason to consider one of the density estimations more accurate than another, but it is tempting to associate the value by which they disagree with the uncertainty of the estimates obtained by these simplified models. All models predict densities that are within a factor of 2 of each other, which is generally comparable to the uncertainty introduced into the inversion process by the background magnetic field models or cross-phase analysis used to calculate the frequencies from the magnetometer data.

![Fig. 1. Frequencies of the FLRs inferred from CANOPUS magnetometers on February 9, 1995. CGM latitudes are shown for all station pairs.](image-url)
We also note that at both the Eskimo–Fort Churchill and Rankin–Eskimo magnetometer pairs for some MLTs the density predicted by the Singer et al. (1981) and Rankin et al. (2000) models for a poloidal mode is higher than that for toroidal mode. For a pure dipole field, such behavior does not occur, as shown, for example, by Cummings et al. (1969). There is nothing in the Singer et al. (1981) or Rankin et al. (2000) models, however, to enforce a similar restriction for general fields. We emphasize that such behavior occurs only for the highly disturbed field lines which extend far into the magnetotail, and therefore lie well outside the area of applicability of either model. For such field lines a more complicated analysis based on Eqs. (5)–(8) is required and a simple description of a standing shear Alfvén mode as either toroidal or poloidal is no longer possible. Another interesting aspect of the comparison seen in Fig. 2 is that, at the two highest latitudes, the densities obtained by assuming toroidal and
poloidal modes for the Singer et al. (1981) model coincide for some MLTs. If this were physical, it would mean that at these particular locations a standing shear Alfvén wave could have an arbitrary polarization. However, once again we believe this feature to be just an artifact of the Singer et al. (1981) model, since more accurate equations (5)–(8) do not exhibit any unusual behavior at these locations. We also note that the densities for the two modes computed with the Rankin et al. (2000) model also coincide at some MLTs for the Rankin–Eskimo magnetometer pair, although at different locations than those for the Singer et al. (1981) model. We dismiss this behavior as an artifact.

Finally, we would like to point out a spike in the density predicted by the toroidal mode of Eq. (2) at around 13 MLT for the Fort Churchill–Back magnetometer pair. This is clearly an artifact of computing numerically the distance between the two field lines spaced radially, since no corresponding spike appears in the data (see Fig. 1), or in any of the two other models.

All models discussed in this paper are based on ODEs and therefore, require very small computational resources. It may take a few minutes on a typical desktop PC to complete a calculation using the Rankin et al. (2006) model (most of this time is spent evaluating metric tensor components along the field line) as compared to a few seconds for Singer et al. (1981) or Rankin et al. (2000) model. Therefore, it is desirable to have a simple criterion defining whether a more involved calculation is necessary for a particular field line. Unfortunately, it is not easy to formulate such a general criterion. It is, however, clear that magnetic field line torsion is one of the important parameters included in the Rankin et al. (2006) model, but not in the Singer et al. (1981) or Rankin et al. (2000) models. Magnetic field line torsion is given by $\tau = \mathbf{p} \cdot \mathbf{n}'$ where $\mathbf{p} = \mathbf{b} \times \mathbf{n}$ is the unit binormal, $\mathbf{n} = \mathbf{b}'/|\mathbf{b}'|$ is the unit principal normal to the field line, and $\mathbf{b} = \mathbf{B}_0/|\mathbf{B}_0|$ is the unit tangent vector for the field line (e.g. Kreszgix, 1988, Section 8.6). Primes here mean differentiation with respect to the field line arc length $s$. Non-zero torsion of the field line gives rise to the off-diagonal terms in Eqs. (5)–(8); however, their importance also depends on the length of the section of the field line where torsion is significant. Therefore, we propose to use “integrated torsion” defined as integral of the absolute value of torsion over the field line length

$$\tau_{\text{int}} = \int |\tau(s)| \, ds,$$

as a simple measure of importance of the off-diagonal terms in Eqs. (5)–(8). Note that $\tau_{\text{int}}$ is a dimensionless quantity. Fig. 3 shows this quantity for the field lines considered in this work. Not surprisingly, $\tau_{\text{int}}$ increases with latitude. As a function of MLT, it has a minimum around the local noon, which is a location separating the field lines swept back on dusk-side from those swept back on dusk-side. Therefore, the torsion of the magnetic field lines at this location would be relatively small. In fact, in this region, even for the highest latitudes the Rankin et al. (2006) model results are quite close to those obtained with Singer et al. (1981) model. It appears that if $\tau_{\text{int}} \lesssim 2$, the approximate equations give the density values which are within 10–20% of the more accurate estimates. Therefore, for $\tau_{\text{int}} \lesssim 2$, the approximate equations are generally adequate for describing frequencies of the FLRs. However, this simple criterion is certainly imperfect, and most likely the question of differences between various models may be answered only by actually solving the corresponding equations and comparing the results.

Finally, we discuss the polarizations of the two modes computed with the approach based on Eqs. (5)–(8). Fig. 4 shows the polarizations (in radians) obtained from the magnetic perturbation vectors at the northern ionosphere and equatorial polarizations based on the electric field perturbations. Toroidal direction at either ionosphere or the equator is defined as azimuthal direction (with respect to the dipole axis). In Fig. 4, a polarization of 0 corresponds to a purely toroidal direction, and a polarization of $\pi/2$ to purely poloidal direction. Thus, what is referred to as toroidal mode through this paper will have ionospheric polarization based on $\delta B$ of 0, and equatorial polarization based on $\delta E$ of $\pi/2$, the opposite holds for poloidal mode. We note that there is some arbitrariness in the definition of the magnetic equator; here we defined it as the point on the field line furthest from the Earth, which is generally very close to the minimum of $B_0$ along the field line.

At lower latitudes, the two modes can be clearly identified as either poloidal or toroidal, but this distinction becomes less clear at higher latitudes. Equatorial polarizations, based on the electric field of the wave, remain more clearly poloidal or toroidal for latitudes of about 67.5° (Back–Gilliam magnetometer pair) than the ionospheric polarizations based on the magnetic field of the wave. It is worthwhile pointing out, however, that the polarization at the ionosphere may not straightforwardly indicate the polarization at the magnetic equator for this model. At the two highest latitudes used in our study neither definition of
the polarization clearly classifies the modes as either poloidal or toroidal; instead both modes have mixed characteristics. It can also be noted that, for some reason, the poloidal mode generally seems to have a polarization at the ionosphere which hovers around $p = 2$ for all MLTs (except for the Rankin–Eskimo magnetometer pair where, at about 18 h MLT, it suddenly deviates from $\pi/2$ value), while the other mode, which can be loosely identified as toroidal, has a polarization that often varies over the whole possible range. Similar findings were also pointed out by Rankin et al. (2006). We note that, on the flanks of the magnetosphere, both modes often seem to have polarizations close to $\pi/2$. The location where the toroidal mode has a polarization close to 0 is always located at the dayside magnetosphere in the region of small torsion of the magnetic field lines, as indicated by Fig. 3. This would be the location where the cross-coupling terms (quantified by the off-diagonal component of the metric tensor $g^{12}$) in
Eqs. (5)–(8) are less important than on the flanks of the magnetosphere. It is therefore reasonable to expect that at these locations the two modes will be nearly decoupled and will have orthogonal polarizations.

4. Conclusions

In this paper we have compared three different eigenproblem models (Singer et al., 1981; Rankin et al., 2000, 2006) for computing the frequencies of the field line resonances and described their effects on the computation of magnetospheric densities from ground-based magnetometer measurements. As ground-based magnetometer chains become more extended and data analysis techniques more refined, remote sensing of the magnetosphere using these data becomes more common, which makes it necessary to assess all the uncertainties inherent in the various techniques used for these purposes. There are several causes of uncertainties associated with modeling FLRs. These include background magnetic field, plasma density parameterization, and FLR models. The first two of these causes have been investigated on many occasions in the past. In this paper we examined the last of these for the first time. We discuss the limitations inherent in the three available eigenproblem models which are most suitable for “magnetosismology” applications and find that for a quiet day, for CGM latitudes below $\approx 68^\circ$ all models give virtually the same results. Therefore, up to these latitudes it is possible to use the simplest of the eigenproblem models, namely that of Rankin et al. (2000) for data inversion. We furthermore suggest a simple, although imperfect, criterion based on the torsion of the field line which can be used to estimate whether simpler models of Singer et al. (1981) or Rankin et al. (2000) are applicable. We find that around magnetic noon, where torsion of the field lines is smaller than in the flanks of the magnetosphere, the model of Singer et al. (1981) appears to work well even at higher CGM latitudes, corresponding to the most northern magnetometers of the Churchill line. In general, at these latitudes, however, it is preferable to use a more realistic FLR model based on covariant–contravariant description of the shear Alfvén waves (Rankin et al., 2006). Although somewhat more computationally demanding and less robust than the other two models the approach of Rankin et al. (2006) has the advantage of computing the polarization of the waves while in other models the polarization has to be assumed beforehand. We find that the polarizations computed self-consistently may differ significantly in the flanks of the magnetosphere from the pure toroidal or poloidal polarizations commonly assumed in the analysis based on simplified equations. This fact may be important in the future interpretation of the ground-based magnetometer data. For the high CGM latitudes (above $68^\circ$) we find that plasma density estimated with different FLR models may differ by a factor of 2. This is comparable to the uncertainty associated with other aspects of the problem (e.g. background magnetic field and density distribution along the field lines).

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