Abstract. The spectral difference in ULF wave amplitude between closely spaced meridional ground stations may be used to measure the eigenfrequency of magnetospheric field lines [Baransky et al., 1985]. A more reliable technique based on the crossphase spectrum has been used to identify eigenfrequencies and study the temporal evolution of local field line resonances. P\(c_3\) (22-100 mHz) pulsations recorded with two pairs of low latitude ground stations have been specifically examined. Resonances and harmonics whose frequencies are in excellent agreement with model calculations of standing toroidal field line resonances, when ionospheric mass loading at low latitudes is taken into account, were identified virtually every day. This points to a diagnostic technique for monitoring temporal variations in ionospheric and magnetospheric plasma parameters.

Introduction

Oscillations of the geomagnetic field with frequencies in the range 22-100 mHz, called P\(c_3\) pulsations, are seen virtually daily at low latitudes, with amplitudes typically \(<1\text{ nT}\). Dungey [1954] first suggested these were due to hydromagnetic (hm) waves in the magnetosphere. In a cold plasma permeated by a uniform magnetic field these waves propagate in the Alfvén and fast modes. The fast mode wave propagates isotropically and distributes energy throughout the magnetosphere. Southwood [1974] and Chen and Hasegawa [1974] showed that these modes could interact to generate azimuthal standing wave oscillations on geocentric field lines. The resonant and harmonic frequencies depend on the length of the field line, the magnitude of the geomagnetic field and the magnetospheric plasma density distribution along the field line. These resonances drive Pedersen and Hall currents in the ionosphere that are detected on the ground as variations of the geomagnetic field in the north-south direction [Hughes and Southwood, 1976].

Experimental evidence for toroidal field line resonances is based on the observation of wave characteristics analogous to a resonant stretched string [eg. Sugliura and Wilson, 1964; Green, 1978]. Furthermore, in situ observations have identified harmonic structure in spectral density and variation in frequency with radial distance which is also consistent with Alfvén mode field line resonances [eg. Takahashi et al., 1984; Engebretson et al., 1986; Fraser et al., 1990]. However, one outstanding problem is the identification in ground data of the particular field line sustaining the resonance. The identification of resonance characteristics usually requires an array of ground stations, and is not straightforward [eg. Hanson et al., 1979]. The effects of spatial integration, wave mode coupling, and the general complexity and variability of spectral features in pulsation records [Menk, 1988] hinder the interpretation of field line resonance features. Furthermore, at low latitudes harmonic frequencies are not integer multiples [Poulter et al., 1988].

Considering these difficulties, Baransky et al. [1985] proposed the amplitude gradient method for measuring the resonant frequency, \(\omega_\text{k}\), at a particular latitude for individual pulsation events. This requires a precise determination of the pulsation signature at two ground stations separated in latitude by a relatively small distance \((\leq 100 \text{ km at } L=2.8)\). We found this method to be sensitive to data selection and have therefore evaluated an alternative technique based on the crossphase difference between the two stations which highlights only those signals exhibiting a latitudinal variation. We find that at low latitudes resonances measured in this way compare favourably with toroidal mode eigenfrequency calculations of Poulter et al. [1988].

Evaluation of the Resonant Frequency

We consider standing transverse field line oscillations excited by an incoming fast mode hm wave in the P\(c_3\) range. Assuming that (i) the fast mode driving wave is monochromatic; (ii) magnetic field lines are uncoupled; (iii) only damping due to ionospheric dissipation is present; and (iv) coupling between the two wave modes is the same for all field lines, the field line resonance may be described by the equation for forced damped simple harmonic motion [Gough and Orr, 1984]:

\[
\ddot{b}_\phi + 2\gamma\dot{b}_\phi + \omega^2 b = \omega_0^2 c \sin \omega t
\]  

where \(b\) is the Alfvén mode wave magnetic field, \(b\) the fast mode wave magnetic field, \(\omega_0\) the frequency of the incoming fast mode driving wave, \(\omega\) the resonant frequency of the magnetic field line, \(c\) the coupling constant between the two modes and \(\gamma\) the damping term.

We consider field line oscillations monitored by two ground stations separated in latitude by ~80 km. The assumption of a single monochromatic fast mode driving wave can be relaxed without affecting the model. Suppose the fast mode is the sum of monochromatic waves of different frequencies, so that each field line is stimulated independently at its eigenfrequency. The northern station detects a resonant frequency \(\omega_{\text{R}}\), while a slightly different frequency, \(\omega_{\text{S}}\), is present in the spectrum at the southern station. The amplitude response as a function of driving frequency, \(A(\omega)\) is shown in Figure 1(a). The spectral difference shown in Figure 1(b) was called the amplitude gradient by Baransky et al.
Fig. 1 Schematic plots for two damped resonant systems with slightly different eigenfrequencies. (a) The amplitude response \( A(\omega) \) in each case. (b) The meridional amplitude difference \( A(\omega_1) - A(\omega_2) \). (c) The meridional crossphase difference \( \phi(\omega_1) - \phi(\omega_2) \). In (b) the resonant frequency \( \omega_0 \) is determined by the condition \( A(\omega_0) - A(\omega) = 0 \) and in (c) where \( \phi(\omega_0) - \phi(\omega) \) is a maximum.

[1985]. We also consider \( \phi(\omega_1) - \phi(\omega_2) \), the crossphase difference, shown in Figure 1(c).

Baransky et al. [1985] used the amplitude difference to determine resonant structure in mid-latitude pulsation data. The resonant frequency was identified as the frequency where the amplitude difference equalled zero with a negative slope as shown in Figure 1(b). On applying this method to several days of Pc3 data at \( L=1.8 \) we found it difficult to unambiguously identify the resonant frequency. In all cases there was more than one frequency satisfying the above conditions. Relatively few cases showed clear positive and negative peaks similar to Figure 1(b). Sometimes \( A(\omega_1) \) and \( A(\omega_2) \) were the same over the entire 10-100 mHz range, suggesting identical \( H \) component signals at both stations. At \( L=2.7 \) the amplitude difference became even more difficult to interpret.

An alternative method is to evaluate the crossphase between the \( H \) components for the two closely spaced stations to obtain the crossphase difference function, following Figure 1(c). The peak in the crossphase spectrum identifies the resonance. The crossphase may be calculated from the output of an FFT and requires minimal extra computing. Determining the resonant frequency in this way incorporates all the advantages of crossphase analysis, with its relative insensitivity to variations in amplitude and the rejection of incoherent frequency components [Waters et al., 1991].

**Experimental Method and Results**

To investigate the amplitude and crossphase difference methods for low latitude Pc3 pulsations two pairs of identical recording stations were established. Stations in each pair were located 80 km apart along the 228° geomagnetic meridian, with one pair centered at \( L=1.8 \) (Gloucester and Newcastle) and the other at \( L=2.7 \) (Launceston and Lommont). Details are given in Table 1. Instrumentation at each site comprised a single component (geomagnetic north-south, \( H \)) induction magnetometer with identical amplitude, frequency and phase response over the 10-100 mHz range.

Data were sampled at 0.5 Hz and recorded with a resolution of 0.03 nT and a time error of <30 ms. Data from the \( L=1.8 \) pair recorded over 4 months in 1989 and from all 4 stations from 14 October to 7 November, 1990, have been examined.

### TABLE 1. Geographic and geomagnetic coordinates of the recording stations.

<table>
<thead>
<tr>
<th>Station</th>
<th>Geographic</th>
<th>Geomagnetic</th>
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<tbody>
<tr>
<td>GLO</td>
<td>32.1°S</td>
<td>150.0°E</td>
</tr>
<tr>
<td>NEW</td>
<td>32.6°S</td>
<td>151.7°E</td>
</tr>
<tr>
<td>LAU</td>
<td>41.7°S</td>
<td>147.1°E</td>
</tr>
<tr>
<td>LEM</td>
<td>42.3°S</td>
<td>147.5°E</td>
</tr>
</tbody>
</table>

The use of the crossphase difference technique in monitoring the temporal and spatial characteristics of low latitude field line resonances is illustrated with a representative example. Figures 2(a) and (c) show dynamic power spectra at GLO \( (L=1.8) \) and LAU \( (L=2.7) \) on 17 October, 1990. These are the northern stations of each pair. The power spectra from the respective southern stations were identical counterparts. \( K_p \) during this day averaged 2+. Figures 2(b) and (d) show the dynamic crossphase spectra between the nearby station pairs. Comparison with the power spectra reveals several characteristics of the field line resonance. The \( L=1.8 \) spectra show that although power at the resonant frequency in Figure 2(a) decreased by 20dB between 0645-0845, the crossphase analysis continued to identify the resonant structure. Figure 2(c) is a typical \( L=2.7 \) power spectrum, with most of the power in the 10-20 mHz range. Figure 2(d) shows that while this band exhibited resonant characteristics and corresponds to the lowest observed harmonic,
The data presented are typical examples showing the temporal variation in resonant structure at \( L=1.8 \) and \( L=2.7 \). The \( L=1.8 \) field line exhibits resonance oscillations under almost all daytime conditions even in the absence of discrete structures in the power spectrum. This implies the observed resonance signature can be generated by background ULF noise in the magnetospheric cavity as well as discrete hm waves. In 42 days of data examined, there were only 2 days when no resonance signature was identified with the \( L=1.8 \) station pair. \( K_p \) for these two days averaged 0+. Harmonics at \( L=1.8 \) have not yet been observed. It is considered that the observations presented form the ground counterpart of recent AMPTE/CCE satellite measurements of harmonic structure over \( L=2-6 \) reported by Takahashi et al. [1990]

To investigate spatial dependence of the resonant structure a third magnetometer was operated at Kulnurra (KUL), 80 km south of NEW, during 1989, giving a GLO-KUL spacing of 160 km. The GLO-NEW and NEW-KUL crossphase spectra had the same overall shape, with the frequency ~3 mHz lower for the NEW-KUL pair. These observations confirm that the resonance region has a ground latitudinal extent at \( L=1.8 \) of \( \pm 80 \) km (\( \pm 0.03 \)). This compares with ionospheric values of 20-40 km for \( P_c3 \) at \( L=1.9 \) [\( \Delta L=0.02 \); Lathuillere et al., 1981]. An identical magnetometer was also operated at a site 100 km west of GLO. Crossphase analysis between the azimuthal stations showed no evidence of resonant structure, even when this was observed between the meridional stations. This indicates a high degree of wave mode decoupling in the magnetosphere.

We next compare our empirical resonant and harmonic frequencies with those determined using the toroidal mode model of Poulter et al. [1988]. These were calculated for local noon including \( E \times B \) drifts, and are listed in Table 2. These calculations identify the observed LAU-LEM frequencies as the fundamental, second and third harmonics. We have estimated the error in our experimental results by considering the ~3dB bandwidth of the crossphase difference peak. The observed results are within experimental error of the model calculations.

During early local morning the temporal variation of the resonance structure shown in Figure 2 exhibits decreasing frequency at \( L=1.8 \) but constant frequency in all three bands at \( L=2.7 \). Sunrise and sunset times on the ground at \( L=1.8 \) on this day were 0512 and 1810 LT. This early morning effect was seen on 14 of 16 days at \( L=1.8 \) but on only 2 days at \( L=2.7 \). These observations suggest that the ionosphere is influencing the resonance structure. Bailey [1983] calculated the diurnal variation of ion density at \( L=1.4 \) and showed that \( O^+ \) concentration increases by a factor of three over 05-07 LT, remains approximately constant until 18 LT, then decreases over 18-20 LT. Since the eigenfrequency of a magnetic flux tube is inversely proportional to the plasma density [Orr, 1984], the observed early morning frequency decrease may reflect the increasing ion concentration. This would be more significant at \( L=1.8 \), where more of the field line is in the

<table>
<thead>
<tr>
<th>Geomagnetic latitude</th>
<th>Experimental (mHz)</th>
<th>Poulter et al. [1988] (mHz)</th>
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<tbody>
<tr>
<td>42.0° (GLO-NEW)</td>
<td>50 ± 2</td>
<td>49 (1)</td>
</tr>
<tr>
<td>53.5° (LAU-LEM)</td>
<td>15 ± 2</td>
<td>16 (1)</td>
</tr>
<tr>
<td></td>
<td>41 ± 2</td>
<td>40 (2)</td>
</tr>
<tr>
<td></td>
<td>62 ± 2</td>
<td>62 (3)</td>
</tr>
</tbody>
</table>

Discussion

The observed results are within experimental error of the model calculations.
ionosphere, than at L=2.7. An evening increase in frequency is usually absent as Pc3 activity disappears around 18 LT. We expect this diurnal variation to be seasonally dependent.

Conclusions

Previous ground studies show that field line resonance characteristics are not always obvious when examining Pc3-4 data solely with power spectrum techniques. The crossphase difference between the H components of nearby meridional stations is a more reliable monitor of the resonant structure, being less sensitive to amplitude variations. Low latitude field lines appear to be stimulated virtually continuously during local daytime by ULF activity in the magnetospheric cavity. The resonance must have a relatively high Q in order to be detected on the ground at L=1.8 with a station spacing of 80 km. The observed resonant frequencies are in excellent agreement with toroidal mode model calculations, and although harmonic structure is often observed at L=2.7 it has not yet been found at L=1.8. The crossphase difference technique has the potential to be used as a diagnostic monitor of temporal variations in local field line resonance, and hence provide a measure of plasma density variations in the plasmasphere and magnetosphere.

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